## Mode-based Reactive Synthesis

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**Abstract.** Reactive synthesis aims to automatically generate systems from high-level formal specifications, but its inherent complexity limits its scalability to real-world scenarios. This limitation can be addressed by decomposing the specification into independent parts for parallel synthesis, but the dependency between variables limits this approach.

At the same time, specifications used in Requirements Engineering (RE) often include high-level state machine descriptions, known as modes, which structure the specification.

This paper introduces a novel method for the sequential decomposition of reactive synthesis problems based on modes. Our approach automatically uses modes to break down a specification into smaller sub-specifications, synthesizes each independently, and then integrates the solutions into a cohesive global model. We present an algorithm that exploits mode transitions and ensures consistency across synthesized components leveraging off-the-self reactive synthesis tools.

We prove the correctness of our approach and show empirically that our method significantly improves scalability when decomposing real-world specifications, outperforming state-of-the-art monolithic tools. As the first sequential decomposition approach, our method offers a promising alternative for scalable reactive synthesis.

## 1 Introduction

Reactive systems [47], which continuously interact with their environment, are essential in domains like cyber-physical and embedded systems. These systems are crucial for tasks such as model checking [19], property monitoring [8], and model-based testing [30]. Linear-Time Temporal Logic (LTL) [56] is commonly used to specify properties of reactive systems, typically in an assume-guarantee format  $(A \to G)$ . Here, Assumptions (A) describe the uncontrollable environment and Guarantees (G) define the desired system behavior. This separation into environment-controlled and system-controlled variables facilitates effective analysis and synthesis [11].

Reactive synthesis automates the construction of a *controller* from a specification, ensuring that for all valid environment inputs, the controller behaves as required. Despite progress in the field, synthesizing controllers for complex

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specifications remains computationally challenging. Even with efficient LTL fragments like GR(1) [55,10], deciding realizability and generating a controller can lead to exponential blow-ups [38]. Several decomposition techniques have been proposed [27,43,7,20,52] which try to perform synthesis independently for different variables, but these methods face difficulties when sub-specifications share controllable variables. Requirements Engineering (RE) methodologies provide useful mechanisms to modularize and specify the system behavior. A typical way to organize system requirements is through the use of high-level state machine descriptions in which the states, referred to as modes, encapsulate specific system behavior under particular situations and transitions represent how the system execution evolves and react to environmental events [37].

Mode-based synthesis leverages these modes to synthesize complex systems by decomposing global specifications into sub-specifications for individual modes. However, the existing approach [13] requires manual intervention, limiting their automation. In this work, we build upon these ideas to fully automate the modebased synthesis process, eliminating the need for manual engineering input. Our approach takes as input the system's global specification, a description of its modes, and optionally, the transitions between them. Through a process we call sequential decomposition, the synthesis problem is addressed incrementally: each sub-problem is solved independently, focusing on one mode at a time. A key aspect of our method is the treatment of initial conditions for each mode. These conditions ensure coherence between sub-specifications and the global one, facilitating seamless connections across modes. By maintaining alignment with global requirements, our method guarantees consistency across mode transitions, minimizing potential conflicts and ensuring the system's correctness. We formalize mode projection and mode-based synthesis, proving consistency between modes and the global system. Crucially, our approach computes initial conditions ensuring that if each sub-specification is realizable, then the global specification is also realizable. The resulting structured controllers enhance transparency and interpretability while improving synthesis scalability, as shown empirically against state-of-the-art monolithic tools. The paper is organized as follows: Section 2 covers preliminaries, Section 3 details our approach, Section 4 presents empirical results, and Section 5 concludes.

**Related Work.** Reactive synthesis [57,10] aims to automatically generate correctby-construction controllers from temporal logic specifications. LTL synthesis is 2EXPTIME-complete [57], tractable fragments like GR(1) enable polynomial-time 75 synthesis [10]. However, challenges remain, particularly in constructing determin-76 istic automata for large Safety-LTL formulas [70]. Compositional approaches im-77 prove scalability by decomposing synthesis tasks [60,57]. Dureja and Rozier [25] 78 reduce model-checking tasks via dependency analysis, while Finkbeiner et al. [29] 79 extend this idea to synthesis by focusing on *controllable* variables. However, 80 81 these simultaneous decomposition methods, which address the synthesis problem in parallel, struggle when requirements share many controlled variables, limiting their applicability [39,29,51]. In RE, high-level state machines, or modes,

are commonly used to structure system specifications [33,64,65,59,4]. Languages like EARS [50], SCR [36,35], TLSF [40], SPIDER [41], NASA's FRET [31], 85 and Spectra [48] leverage modes for organizing computation and requirements. This aligns with IEEE standard 29148, which notes that "some systems behave 87 quite differently depending on the mode of operation. For example, a control system may have different features depending on its mode: training, normal, or emergency." [1]. State-based techniques like Statecharts [34], Broy's hierarchical service modeling [15], and the SCR method further formalize mode-based 91 specifications. Feature modeling [24,61] and safety analysis methods like FTA 92 and FMEA [44,2] also utilize modes. However, translating these mode-based specifications into LTL can increase complexity, hindering simultaneous methods methods. For example, NASA's FRET, which heavily uses state variables (modes), poses challenges for decomposition tools, leading to issues like false positives [51] and goal conflicts [21,16,12,22,49,11]. This motivates the need for more efficient synthesis approaches in real-world applications. Recent RE research explores using Large Language Models (LLMs) for requirements specification [67,5,54,69,68,66]. LLMs, like GPT series [14,53], LaMDA [62], LLaMa [63], 100 PaLM [18], and BERT [23], can assist in articulating mode-based requirements, potentially simplifying translation to LTL [45,3]. Related to us, Balkan et al. [6] 102 use modes in a GR(1) subfragment for control design of continuous systems, 103 focusing on quantitative performance. This contrasts with our focus on discrete 104 systems and logical correctness in reactive synthesis. More directly, Brizzio et 105 al. [13] proposed a mode-based decomposition for a fragment of safety, but re-106 quire manual specification of initial conditions, which is tedious, error-prone, and 107 deviates from standard RE practices. In contrast, our novel mode-based synthesis automatically generates initial conditions, ensuring consistency and eliminating 109 manual intervention. Unlike simultaneous decomposition, we employ a sequen-110 tial approach. By addressing one sub-problem at a time and leveraging natural 111 mode transitions, our sequential method simplifies synthesis, reduces potential 112 conflicts, and inherently ensures consistency. To the best of our knowledge, this 113 is the first fully automated sequential mode-based synthesis method.

### 2 Preliminaries

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LTL is a logical formalism widely used to specify reactive systems [56,46]. Given a set of propositional variables AP, LTL formulas are defined using standard logical connectives and the temporal operators  $\bigcirc$  (next) and  $\mathcal{U}$  (until) as follows:

$$\varphi ::= \mathtt{true} \; \big| \; a \in \mathsf{AP} \; \big| \; \varphi \vee \varphi \; \big| \; \neg \varphi \; \big| \; \bigcirc \varphi \; \big| \; \varphi \; \mathcal{U} \; \varphi$$

Other common operators, such as false,  $\wedge$  (and),  $\square$  (always), and  $\rightarrow$  (implies), can be derived:  $\phi \wedge \psi \equiv \neg(\neg \phi \vee \neg \psi)$ ,  $\square \phi \equiv \neg(\mathsf{true} \mathcal{U} \neg \phi)$ , and  $\phi \rightarrow \psi \equiv \neg \phi \vee \psi$ . Given an LTL formula  $\varphi$ ,  $Vars(\varphi) \subseteq \mathsf{AP}$  denotes the set of atomic propositions used in  $\varphi$ . The semantics of LTL associate traces  $\sigma \in \Sigma^{\omega}$  with formulae as follows (we omit the Boolean operators which are standard):

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\sigma \vDash a iff a \in \sigma(0)

\sigma \vDash \bigcirc \varphi iff \sigma^1 \vDash \varphi

\sigma \vDash \varphi_1 \mathcal{U} \varphi_2 iff for some i \ge 0 \sigma^i \vDash \varphi_2, and for all 0 \le j < i, \sigma^j \vDash \varphi_1
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where  $\sigma(i)$  is the *i*-th letter of  $\sigma$ , and  $\sigma^i \in \Sigma^\omega$  is its suffix starting at the *i*-th position. Given  $L \subseteq \Sigma^\omega$  and a formula  $\varphi$ , we use  $L \vDash \varphi$  if for all  $\sigma \in L$ ,  $\sigma \vDash \varphi$ .

A Syntactic Fragment of Safety-LTL. We focus on a fragment of LTL 126 (Safety-LTL) [70] commonly used in requirement engineering. LTL<sub>G</sub> consists of formulas of the form  $\psi \wedge \Box \varphi$ , where  $\psi$  is propositional and  $\varphi \in LTL_X$  (which 128 uses only the  $\bigcirc$  operator). LTL<sub>G</sub> is widely used in industrial safety specifications [17,28,32]. Specifically, we work with  $\mathsf{GX}_0$  [13], a sub-fragment of Safety-130 LTL defined as  $\alpha \to (\beta \wedge \Box \psi)$ , where  $\alpha, \beta$ , and  $\psi$  are conjunctions in LTL<sub>X</sub>. 131 This fragment extends  $LTL_G$ , still expresses safety properties, and is supported 132 by tools like Strix [52]. A reactive specification  $\varphi = (A, G)$  consists of  $A = (\theta_e, \varphi_e)$ and  $G = (\theta_s, \varphi_s)$ , where  $\theta_{\{e,s\}}$  represent initial conditions for the environment 134 and system, respectively, and  $\varphi_{\{e,s\}}$  are the assumptions and guarantees. Like previous works [29,51] that restrict their methods to LTL fragments for effi-136 ciency, we simplify our approach by using propositional formulas over  $Vars(\varphi)$ 137 for A to ensure a consistent environment during decomposition, avoiding inter-138 ference that leads to false-negatives [51], and employ  $G \in LTL_X$  for guarantees. 139 Thus, the intended meaning of  $\varphi$  is the  $\mathsf{GX}_0$  formula:  $(\theta_e \to (\theta_s \land \Box(\varphi_e \to \varphi_s)))$ 140

Reactive Synthesis. Reactive LTL synthesis [57] is the problem of auto-141 142 matically constructing a system that reacts to the environment guaranteeing an LTL specification  $\varphi$ . The propositions  $Vars(\varphi)$  are partitioned into  $\mathcal{X} \cup \mathcal{Y}$ , 143 where  $\mathcal{X}$  are environment-controlled variables and  $\mathcal{Y}$  are system-controlled vari-144 ables. A system strategy for  $\varphi$  is a function  $\rho:(2^{\mathcal{X}})^+\to 2^{\mathcal{Y}}$  mapping fi-145 nite sequences of  $\mathcal{X}$  valuations to  $\mathcal{Y}$  valuations. Given an infinite sequence 146  $X = X_1, X_2, \ldots \in (2^{\mathcal{X}})^{\omega}$ , the play induced by strategy  $\rho$  is the infinite sequence 147  $\sigma_{\rho,X} = (X_1 \cup \rho(X_1))(X_2 \cup \rho(X_1, X_2))\dots$  We use  $\mathcal{L}(\rho) = \{\sigma_{\rho,X} \mid X \in (2^{\mathcal{X}})^{\omega}\}$  for 148 the set of plays played according to  $\rho$ . A play  $\sigma$  is winning if  $\sigma \models \varphi$ . A strategy 149 is winning if  $\mathcal{L}(\rho) \vDash \varphi$ . Realizability is the problem of deciding if a specification 150 has a winning strategy, and synthesis is the problem of computing one.

## 3 Mode-Based Synthesis for $\mathsf{GX}_0$

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We now describe our mode-based reactive synthesis method for  $\mathsf{GX}_0$  specifications, beginning with some preliminary definitions. A mode m for a  $\mathsf{GX}_0$  formula  $\varphi$  is a predicate over  $Vars(\varphi)$  describing a set of system states. A set of modes  $M = \{m_0, \ldots, m_k\}$  is valid for  $\varphi$  if (1) modes are mutually exclusive  $(m_i, m_j \in M \text{ with } i \neq j, m_i \land m_j \text{ is unsatisfiable})$  and (2) they cover all possible states  $(\bigvee_{i=0}^k m_i \text{ is valid})$ . A mode-transition from m to a different mode n in a

trace  $\sigma$  occurs at index i if  $\sigma(i) \vDash m$  and  $\sigma(i+1) \vDash n$ . A mode transition from m to n occurs under a strategy  $\rho$  if it occurs in some trace  $\sigma \in \mathcal{L}(\rho)$ .

Definition 1 (Mode-Graph). A mode-graph is a directed graph  $\mathcal{G} = (M, \prec)$ , where  $M = \{m_0, m_1, \ldots, m_k\}$  is a set of modes, and  $\prec \subseteq M \times M$  is irreflexive.

The intended meaning of  $\prec$  is to restrict the search of strategies to those where

mode-transitions are included in  $\prec$ . In a complete mode graph,  $\prec = \{(m, n) \mid m, n \in M, m \neq n\}$  includes all possible strategies. In RE, it is common to specify modes and mode-transitions as part of the requirements, as discussed in Section 1 and 2.

# 3.1 Basic Mode-Based Synthesis with Initial Conditions

Given a mode  $m \in M$  and a set of guarantees  $\varphi_s$  in a  $\mathsf{GX}_0$  specification  $\varphi$ , the following function reduce projects a new set of guarantees specific to mode m:

$$\operatorname{reduce}(\psi,m) = \begin{cases} \operatorname{true}, & \text{if } (m \wedge \neg \psi) \vDash \operatorname{false} \ (1) \\ \operatorname{false}, & \text{if } (m \wedge \psi) \vDash \operatorname{false} \ (2) \\ \operatorname{Simpl}(\operatorname{reduce}(\psi_1,m) \bullet \operatorname{reduce}(\psi_2,m)), & \text{if } \neg (1 \vee 2) \wedge \psi = \psi_1 \bullet \psi_2 \\ & \text{with } \bullet \in \{\wedge,\vee,\rightarrow\} \end{cases}$$
 
$$\operatorname{Simpl}(\neg \operatorname{reduce}(\psi',m)), & \text{if } \psi = \neg \psi' \\ \square \operatorname{Simpl}(\operatorname{reduce}(\psi',m)), & \text{if } \psi = \square \psi' \\ \bigcirc \operatorname{Simpl}(\operatorname{reduce}(\psi',m)), & \text{if } \psi = \bigcirc \psi' \\ \psi, & \text{otherwise} \end{cases}$$

Given  $\varphi_s$ , reduce  $(\varphi_s, m)$  projects <sup>1</sup> the set of guarantees  $\varphi_s$  on mode m. We use  $\mathsf{Simpl}(\psi)$  for a function that applies standard Boolean simplifications, like  $(x \land \mathsf{true}) \mapsto x$ , etc. The reduced specification for mode m is denoted  $\varphi_m$ .

Example 1. Consider the following  $\mathsf{GX}_0$  specification  $\varphi$  and modes  $m_1$  and  $m_2$ :

$$G_1: \Box(e_1 \to (m_1 \to \bigcirc s_1)), \quad G_2: \Box(m_2 \to \bigcirc(\neg s_3 \lor m_1)), \quad G_3: \Box(\neg m_2 \to s_3)$$

Applying reduce  $(\varphi, m_1)$  results in:

$$G_1: \Box(e_1 \to \bigcirc s_1), \quad G_2: \Box true, \quad G_3: \Box s_3$$

 $G_2$  simplifies to  $\Box$ true, and  $G_3$  simplifies to  $\Box s_3$  after replacing  $m_2$  with false.

The key-stone of mode-based synthesis is that one can focus on simplified specifications for each of the given modes independently—which improves the scalability of off-the-shelf reactive synthesis tools. However, during an execution, a system can transition between different modes. In these transitions, the system may leave the satisfaction of sub-formulas pending for the arriving modes. We call these sub-formulas pending obligations. The following definition captures a subset of them that a mode may leave pending for a successor mode.

Definition 2 (Pending Obligations). The set of (potential) pending obligations for mode  $m_i$  is  $\mathcal{O}^p(m_i) = \{ \psi \mid \bigcirc \psi \in \mathsf{subformulas}(\varphi_{m_i}) \}$ .

<sup>&</sup>lt;sup>1</sup> Throughout the paper, we use "reduced spec." and "projection" interchangebly.

Cumulative Obligations. Cumulative obligations, denoted  $\mathcal{O}^c(m_j)$ , represent the obligations a mode  $m_j$  inherits from its predecessors during transitions. They ensure that all pending requirements are satisfied throughout the system. A straightforward, though imprecise, method to compute them is by aggregating obligations from all predecessors. Formally, for a mode  $m_j$ :  $\mathcal{O}^c(m_j) = \bigcup_{m_i \prec m_j} \mathcal{O}^p(m_i)$ . The concept of cumulative obligations is illustrated in the following example.

Example 2. Consider a mode-graph with  $M = \{m_1m_2, m_3\}$  and  $m_1 \prec m_2$ ,  $m_2 \prec m_3$  and  $m_3 \prec m_1$ . Let  $e_1$  and  $e_2$  be environment-controlled variables, with the system controlling  $\{s_1, s_2, s_3, s_4, m_1, m_2, m_3\}$ . The specification is:

$$\begin{aligned} & \boldsymbol{G_1}: \Box(e_1 \to (m_1 \to (\bigcirc m_2 \land \bigcirc \bigcirc s_2))) & \boldsymbol{G_2}: \Box(e_2 \to (\neg m_3 \to \bigcirc \bigcirc (\neg s_3 \lor s_1))) \\ & \boldsymbol{G_3}: \Box(\neg e_1 \to (m_2 \to \bigcirc (m_3 \land s_4))) & \boldsymbol{G_4}: \Box(m_3 \to \bigcirc (m_1 \land s_1)) \end{aligned}$$

The reduced specifications for modes  $m_1$ ,  $m_2$ , and  $m_3$  are  $\varphi_{m_1} = \mathbf{G_1} \wedge \mathbf{G_2}$ ,  $\varphi_{m_2} = \mathbf{G_2} \wedge \mathbf{G_3}$ , and  $\varphi_{m_3} = \mathbf{G_4}$ . When the system transitions from  $m_1$  to  $m_2$  while  $e_1$  holds, mode  $m_2$  must fulfill the pending obligation  $\bigcirc s_2$ . The exact obligations, however, depend on the order of events. If  $e_2$  occurs before  $e_1$ , the system must satisfy both  $\neg s_3 \vee s_1$  and  $\bigcirc s_2$ . On the other hand, if  $e_1$  occurs before  $e_2$ , or if both hold simultaneously, the system must satisfy  $\bigcirc (\neg s_3 \vee s_1)$  along with  $\bigcirc s_2$ . If the system is in mode  $m_2$  and  $e_1$  does not hold, the system transitions to  $m_3$  leaving the pending obligation  $m_3 \wedge s_4$ . The pending obligations for  $m_1$ ,  $m_2$  and  $m_3$  are:

$$\mathcal{O}^{p}(m_{1}) = \{m_{2}, \bigcirc s_{2}, s_{2}, \bigcirc (\neg s_{3} \lor s_{1}), \neg s_{3} \lor s_{1}\}$$

$$\mathcal{O}^{p}(m_{2}) = \{\bigcirc (\neg s_{3} \lor s_{1}), \neg s_{3} \lor s_{1}, m_{3} \land s_{4}\}$$

$$\mathcal{O}^{p}(m_{3}) = \{m_{1} \land s_{1}\}$$

From this, the *cumulative obligations* are:  $\mathcal{O}^c(m_1) = \mathcal{O}^p(m_3)$ ;  $\mathcal{O}^c(m_2) = \mathcal{O}^p(m_1)$ ;  $\mathcal{O}^c(m_3) = \mathcal{O}^p(m_2)$ .

However, as mentioned before, this simple aggregation is imprecise and can lead to incorrect results. It fails to capture interactions between obligations arising from mode transitions in execution paths as demonstrated in the following example.

• Example 3. Consider a specification with the following guarantees:

$$\begin{array}{ll} \boldsymbol{G_1}: \square(m_1 \to \bigcirc m_2) & \boldsymbol{G_2}: \square(m_2 \to \bigcirc m_1) \\ \boldsymbol{G_3}: \square(m_1 \to \bigcirc \bigcirc p) & \boldsymbol{G_4}: \square(m_1 \to p) \end{array}$$

Here,  $\mathcal{O}^p(m_1)$  includes  $\{m_2, \bigcirc p, p\}$ , while  $\mathcal{O}^p(m_2)$  only contains  $\{m_1\}$  because  $\varphi_{m_2}$  retains only  $G_2$ . If we propagate obligations naively without accounting for mode interactions,  $\mathcal{O}^c(m_1)$  might incorrectly focus on satisfying only  $m_1$ , neglecting the obligation to satisfy p. This error occurs because  $G_2$  forces  $m_2$  to transition back to  $m_1$  after just one step, leaving p—which originates from the obligation  $\bigcirc p$  that  $m_1$  transferred to  $m_2$ —unsatisfied. Such propagation would compromise the correctness of the specification leading to false negatives.

### Algorithm 1: Fixpoint Algorithm for Cumulative Obligations.

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1 Input: \varphi = (A, (\theta_s, \varphi_s)), \mathcal{G} = (M, \prec)
  2 for m_i \in M do
            \mathcal{O}^c[m_j] \leftarrow \quad \bigcup \quad (\mathsf{obligations}(\mathsf{reduce}(\varphi_s, m_i)) \cup \mathsf{obligations}(\mathsf{reduce}(\theta_s, m_i)))
  4 changed \leftarrow \texttt{true}
       while changed do
  5
            changed \leftarrow \texttt{false}
  6
            for (m_i, m_j) \in \prec \mathbf{do}
                                \bigcup_{\substack{\bigcirc k p \in \mathcal{O}^c[m_i]}} \{\bigcirc^{k-1} p, \bigcirc^{k-2} p, \dots, p\} \text{ do}
   8
                     if u \notin \mathcal{O}^c[m_i] then
   9
                      \mid \mathcal{O}^c[m_i] \leftarrow \mathcal{O}^c[m_i] \cup \{u\} \; ; \; changed \leftarrow \mathsf{true}
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11 return \mathcal{O}^c
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To address these challenges, Alg. 1 systematically computes all *cumulative obli*qations that a mode may need to satisfy, ensuring the global specification is correctly enforced across transitions. This algorithm iterates until a stable set of obligations is computed. Given a valid set of modes  $M = \{m_0, \ldots, m_k\}$ , the cumulative obligations  $\mathcal{O}^c(m_i)$  for each  $m_i \in M$  are the conditions that  $m_i$  may be forced to satisfy based on the pending obligations inherited from all its predecessors. Each  $\mathcal{O}^c(m_i)$  is computed using Alg. 1. Revisiting Ex. 3, Alg.1 iteratively determines that when transitioning from  $m_2$  to  $m_1$ ,  $m_1$  must satisfy both p and itself, resulting in  $\mathcal{O}^c(m_1) = \{p, m_1\}$ . We refer to the set  $\mathcal{O} = \bigcup_{m_i \in M} \mathcal{O}^c(m_i)$ as the universe of obligations across different modes. To systematically explore the universe of obligations, we introduce obligation variables that encode each element within this universe, encoding whether the corresponding obligation is considered. We introduce a fresh variable  $v_{\bigcirc^i \varphi}$  for each formula  $\varphi \in \mathcal{O}$ . We use  $v(\varphi)$  for the variable corresponding to  $\varphi$ . While Alg. 1 performs the correct propagation of obligations, the set of obligations computed is a superset of the obligations that a mode may be requested to fulfill. Asking an instance of a mode to satisfy a larger subset of the cumulative obligations makes the instance more difficult to be realizable, while it helps predecessor instances to be realizable. The concept of *initial condition* captures this notion of combination of obligations.

**Definition 3 (Initial Conditions).** Given a mode  $m_i \in M$ , the set of initial conditions  $\mathcal{I}(m_i)$  is the set of all possible conjunctions of subsets of cumulative obligations:

$$\mathcal{I}(m_i) = \left\{ igwedge_{\phi \in \mathcal{S} \cup \{ extit{true}\}} \phi \;\middle|\; \mathcal{S} \subseteq \mathcal{O}^c(m_i) 
ight\}$$

Example 4. Consider  $\mathcal{O}^c(m_1) = \{\bigcirc q, \bigcirc r\}$  and  $\mathcal{O}^c(m_2) = \{s\}$ . The initial conditions are:  $\mathcal{I}(m_1) = \{\bigcirc q \land \bigcirc r, \bigcirc q, \bigcirc r, \mathsf{true}\}$  and  $\mathcal{I}(m_2) = \{s, \mathsf{true}\}$ .

<sup>&</sup>lt;sup>2</sup> Each  $x \in \mathcal{I}(m_i)$  is conjoined with  $m_i$ . For readability, it is omitted from the text.

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### Algorithm 2: Sequential Decomposition.

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1 Input: \varphi = (A, (\theta_s, \varphi_s)), \Theta = \{\theta_{s_0}, \dots, \theta_{s_n}\}, \mathcal{G} = (M, \prec), \mathcal{O}
   2 \mathcal{O}_{\text{sorted}} = \text{sortBySize}(\mathcal{O},\downarrow) \; ; \; \theta'_{s_i} \leftarrow \texttt{true}
   3 for \psi \in \theta_s do
              \psi' \leftarrow \psi[f \backslash v(f)] \text{ for all } f \in \mathcal{O}_{\text{sorted}}
              \theta'_{s_i} \leftarrow \theta'_{s_i} \wedge \psi'
   6 for m_i \in M do
              \varphi_s' \leftarrow \mathsf{reduce}(\varphi_s, m_i) \; ; \; \varphi_{m_i}^R \leftarrow \mathsf{true}
              for \psi \in \varphi'_s do
                    \psi' \leftarrow \psi[f \backslash v(f)] for all f \in \mathcal{O}_{\text{sorted}}
                   \varphi_{m_i}^R \leftarrow \varphi_{m_i}^R \wedge (\neg done \rightarrow \psi')
 10
              for \bigcirc^k p \in \mathcal{O}_{sorted} do
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                   if k = 1 then \varphi_{m_i}^R \leftarrow \varphi_{m_i}^R \wedge ((\neg done \wedge v(\bigcirc p)) \rightarrow \bigcirc (\neg done \rightarrow p));
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                    else \varphi_{m_i}^R \leftarrow \varphi_{m_i}^R \wedge ((\neg done \wedge v(\bigcirc^k p)) \rightarrow \bigcirc(\neg done \rightarrow v(\bigcirc^{k-1} p)));
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              for (m_i, m_j) \in \prec, and \bigcirc^k p \in \mathcal{O}_{sorted} and \theta_{s_j} \in \Theta do 
 \mid if (\theta_{s_j} \land \neg v(\bigcirc^{k-1} p)) is sat then \varphi_{m_i}^R \leftarrow \varphi_{m_i}^R \land (jump_j \to \neg v(\bigcirc^{k-1} p));
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              \varphi_{m_i}^R \leftarrow \varphi_{m_i}^R \wedge ((\bigvee_{(m_i, m_j) \in \prec} jump_j) \rightarrow \bigcirc done)
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               \varphi_{m_i}^R \leftarrow \varphi_{m_i}^R \land ((\neg \bigvee_{(m_i, m_i) \in \prec} jump_j) \rightarrow (\neg done \rightarrow \bigcirc \neg done))
17
              \Pi[i] \leftarrow (A, (\theta'_{s_i}, \varphi^R_{m_i}))
18
19 return \Pi
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Given a mode-graph  $\mathcal{G}$ , a set of valid modes  $M = \{m_1, \ldots, m_k\}$ , and a  $\mathsf{GX}_0$  specification  $\varphi = (A, G)$ , the mode-based synthesis method consists of generating a set of projections,  $\Pi = \{(\theta_{e_1}, \theta_{s_1}, \varphi_{m_1}^R)^3\}, \ldots, (\theta_{e_k}, \theta_{s_k}, \varphi_{m_k}^R)\}$  such that all are realizable, and can be composed into a strategy for the original specification.

This process must satisfy the following objectives:

- I) Ensure that each projection  $(\theta_{e_i}, \theta_{s_i}, \varphi_{m_i}^R) \in \Pi$  is realizable. Even though the local strategy generated is infinite, it could decide to jump into a successor node and move into a winning sink state.
- II) For each  $m_i \prec m_j$  the successor mode  $m_j$  must satisfy the obligations inherited from its predecessor  $m_i$ , denoted as  $\mathcal{O}^p(m_i) \cap \mathcal{O}^c(m_j)$ . The connection between projections is managed through the initial condition  $\theta_{s_j} \in \mathcal{I}(m_j)$  of the successor  $m_j$ . Formally, for all  $m_i \prec m_j$ ,  $\theta_{s_j} \Longrightarrow \bigwedge(\mathcal{O}^p(m_i) \cap \mathcal{O}^c(m_j))$ . This ensures that a transition from  $m_i$  to  $m_j$  is valid only if  $m_j$  satisfies the pending obligations of mode  $m_i$ , ensuring the correct connection and composition of projections for sequential decomposition.
- 257 III) In the absence of a specific  $\prec$  provided in  $\mathcal{G}$ , the complete graph is used.

Alg. 2 adapted from [13], outlines the mode-based synthesis approach. The algorithm generates a projection  $\varphi_{m_i}^R$  for each mode  $m_i$ , ensuring consistency with **manually provided** initial conditions (as required in point (II)). To demonstrate the method, consider the specification  $\psi$  in Fig. 1. Due to its complexity and the number of variables, traditional synthesis tools often struggle with spec-

<sup>&</sup>lt;sup>3</sup> We explain what the notation  $\varphi_{m_i}^R$  means below

ifications like this. Assume that each mode corresponds to a unique counter value. For instance, in mode  $m_{20}$  (i.e., counter=20), the approach projects the global specification onto  $m_{20}$  relying on manually specified initial conditions. Alg. 2 ensures that the projection  $\varphi_{m_{20}^R}$ , along with others, is consistent with the global specification  $\psi$ . A reduced version of this projection is shown in Fig. 2. Transitions between modes are modeled using fresh variables jump (indicating a transition to a successor mode) and done (indicating that the game for the current mode is completed due to a transition). A transition is allowed only if the initial condition of the successor mode satisfies the obligations at the time of the jump. This guarantees that strategies for different modes are properly connected. Although Alg. 2 significantly improves synthesis speed, it introduces two limitations: (1) Each mode is associated with a single initial condition, and (2) these initial conditions must be manually specified. To address (1), Alg. 2 can be modified to iterate over a set of sets of possible initial conditions  $\Theta = \{\{\theta_{s_i}^1, \dots, \theta_{s_i}^k\}, \{\theta_{s_i}^1, \dots, \theta_{s_i}^k\}, \dots\}$ , updating the jump variable to  $jump_i^i$  for different initial conditions. However, (2) remains a significant challenge due to the considerable manual efenv boolean reset, start;

```
279
        Int(20) counter; sys Int(1) jump
280
     sys boolean trigger, done, o_1;
281
     asm G !(reset and start);
     gar (counter=20 and !done);
283
         G (!done -> trigger):
     gar
     gar G (!done -> counter=20 -> o_1);
     gar G (!done -> reset -> o_1);
285
     gar G (!done and o_1 -> next(done))
           (jump=1 -> next(done));
     gar G (done -> next(done));
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```

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Fig. 2: Reduced specification.

ensuring realizability and consistency.

ever, (2) remains a significant challenge due to the considerable manual effort required to compute initial conditions. To address this, we propose here a method for automatic modebased synthesis, which eliminates the need for manual specification by automatically computing initial conditions for each projection. Alg 2 then generates projections using these automatically computed initial conditions  $\Theta$ ,

```
env boolean reset, start;
sys Int(1..20) counter; sys boolean trigger;
// Start and reset are not initially pressed
asm (!reset and !start);
// Only reset or start can be active at a time
asm G !(reset and start);
// Counter is initially at the lowest value
gar counter=1;
  Restart signal always reset the count
gar G (reset -> next(counter=1) );
   Always stay at the same number or increase it
gar G ( (counter=1 and start) -> next(counter=2 or reset) );
gar G ( (counter=2 and !reset) -> next(counter=3 or reset) );
gar G ( (counter=19 and !reset) -> next(counter=20 or reset) );
gar G (counter=20 <-> trigger);
// Reach the limit and start again
gar G (counter=20 -> next(counter=1));
```

Fig. 1: Counter machine example written in Spectra.

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#### A Fixpoint Search Method for Initial Conditions 3.2

We propose now an automatic mode-based synthesis method that removes the requirement for manual encoding of initial conditions. The challenge is to discover a collection of feasible initial conditions for each mode.

We consider instances of a mode  $m_i$  for each active initial condition  $\theta_{s_i}$ , to support the realizability of predecessors  $m_i$ . Verifying the realizability of the pair  $(\theta_{s_i}, \varphi_{m_i}^R)$  is the main activity, which in turn depends on the active instances of successor modes. This complex circular dependencies between modes and initial conditions requires efficient exploration of the possible instantiations of the mode graph. We solve this problem with a fixpoint search technique that efficiently explores the space of instantiations.

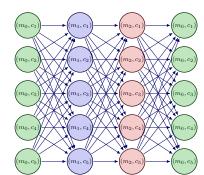
Universe Representation. We start by capturing the representation of the 301 set of possible instantiations.

**Definition 4** (Mode Instance Graph). Given a set of modes M, a universe of initial conditions  $\mathcal{I}(m)$  for each mode m, and a mode graph  $\mathcal{G}=(M,\prec)$ , a Mode Instance Graph (MIG) is a graph  $(V, \mapsto)$ , where:

```
-V \subseteq \{(m,c) \mid m \in M \text{ and } c \in \mathcal{I}(m)\} \land V \neq \emptyset
- \mapsto = \{((m_i, c_j), (m_k, c_l)) \mid (m_i, c_j) \in V, (m_k, c_l) \in V \text{ and } m_i \prec m_k)\}
```

In a mode instance graph, each mode is instantiated with possibly many initial conditions, which are connected to every instance of its successor nodes.

For example, consider a mode-graph with three modes  $m_0$ ,  $m_1$ , and  $m_2$  and  $m_0 \prec m_1 \prec m_2 \prec m_0$ . Assume each mode has five possible initial conditions,



denoted as  $(m_i, c_k)$ . The candidate initial conditions  $c_k$  for each mode  $m_i$  are determined by  $\mathcal{I}(m_i)$ , which includes all possible subsets of the cumulative obligations for that mode (see Def. 3). The universe of exploration in the diagram on the left illustrates the MIG for this simplified example. Our approach systematically evaluates each configuration to ensure that all projections meet the realizability requirements. Informally, our method iterates through the MIG to find a proof of realizabil-

ity for the specification  $\varphi$ , pruning the search space as proofs of realizability are found. Formally:

**Definition 5 (Realizability Proof).** Given an  $MIG = (V, \mapsto)$ , a proof of realizability is a subgraph  $\mathcal{R} = (V', \mapsto')$ , where  $V' \subseteq V$  and  $\mapsto' \subseteq \mapsto$ , such that for every pair  $(m, c) \in V'$ ,  $(c \wedge \square(\varphi_m^R))$  is realizable.

328 Alg. 3 presents our solution, which systematically searches for subsets of the initial conditions, one for each node, aiming to find a realizability proof. The search begins with a candidate proof by creating the MIG that contains all

### **Algorithm 3:** Search for Init. Cond.

```
1 Input: \varphi = (A, (\theta_s, \varphi_s)), \mathcal{G}, M = \{m_0, \dots, m_n\}
  2 \mathcal{O}^c \leftarrow \text{Alg } 1(\varphi, M, \mathcal{G}) ; \mathcal{O} \leftarrow \bigcup_{m \in M} \mathcal{O}^c[m]
3 for m \in \{m_0..m_n\} do \Theta[m] \leftarrow \bigwedge_{\phi \in \mathcal{S} \cup \{\text{true}\}} \phi \mid \mathcal{S} \subseteq \mathcal{O}^c[m] ;
  4 MIG \leftarrow createMIG(\mathcal{G}, \mathcal{I})
  5 do
            initial \leftarrow \mathtt{false} \ ; \ finished \leftarrow \mathtt{true}
   6
            for m \in \{m_0..m_n\} do
   7
                 for each candidate \theta \in \Theta[m] do
   8
                      (A, (\theta, \varphi_m^R)) \leftarrow \text{Alg } 2((A, (\theta, \varphi_s)), \Theta, M[m], \mathcal{G}, \mathcal{O})
   9
                      if realizable(A, (\theta, \varphi_m^R)) then
 10
                        | if m = m_0 then initial \leftarrow \text{true};
 11
                      else
 12
                           finished \leftarrow \texttt{false}
 13
                           MIG \leftarrow MIG \setminus (m, \theta)
 14
                           \Theta[m] \leftarrow \Theta[m] \setminus \theta
 15
       while \neg finished \land \neg initial;
17 return MIG
```

modes and their initial conditions. The algorithm proceeds in rounds, where at each round all remaining instance modes are checked for realizability. Every realizable instance is kept, otherwise (if no successor instantiated with some initial condition can support its realizability), the instance mode is deemed unrealizable and removed. The process iterates until a fixpoint is reached (i.e., a full round is passed with all remaining modes realizable). If all projections are realizable at the fixpoint, the resulting graph is a realizability proof  $\mathcal{R}$ ; otherwise, it is empty.

A minimal modification of Alg. 3, allows us to bypass many realizability checks by inferring results based on previous checks in the same iteration, using memoization [9]. For instance, if a mode with candidate initial condition  $p \wedge q$  is realizable, the same mode will also be realizable for initial conditions p, q and true. Conversely, if p is unrealizable, all initial conditions that imply p are also unrealizable. For example, in our case study (Fig. 1),  $m_1$  spans from 1 to 10, while  $m_2$  spans from 11 to 20. Our algorithm efficiently computes the initial conditions as follows: for  $m_1$ , it deduces the initial condition  $\theta_1 = \text{counter=1}$ , and for  $m_2$ , it derives the initial condition  $\theta_2 = \text{counter=11}$ . These results align with those manually derived in [13].

### 3.3 Correctness

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This section establishes the soundness of the mode-based synthesis algorithm.
We begin by stating a previously established result.

Theorem 1 (Soundness of Alg. 2 [13]). Given a specification  $\varphi$ , a modegraph  $\mathcal{G}$ , and a set of initial conditions  $\mathcal{I}$ , if all projections are realizable, then  $\varphi$  is realizable.

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The following theorem formalizes the soundness of Algorithm 3.

**Theorem 2.** If Algorithm 3 returns a realizability proof  $\mathcal{R}$ , then  $\varphi$  is realizable.

Proof (sketch). We proceed by induction on the length of the environment sequence  $X_1, \ldots, X_i$ . The algorithm constructs a Realizability Proof, where each mode  $m_i$  is associated with a set of possible initial conditions  $\mathcal{I}(m_i)$  ensuring realizability of the projection  $\varphi_{m_i}^R$ . We construct a global winning strategy by composing local winning strategies for each instance.

- Base case: For the initial environment valuation  $X_1$ , the system starts in mode  $m_0$ . The algorithm checks the realizability of  $\varphi_{m_0}^R$  for each initial condition in  $\mathcal{I}(m_0)$ . We can use a winning strategy for  $\varphi_{m_0}$  which provides a winning move in the first step.
- Inductive step: Assume that for the environment sequence  $X_1, \ldots, X_{i-1}$ , all projections  $\varphi_{m_{i'}}^R$  for i' < i are realizable for at least one initial condition in  $\mathcal{I}(m_{i'})$ . Now consider the next input  $X_i$  and the mode  $m_i$ . The algorithm has checked the realizability of  $\varphi_{m_i}^R$  for each initial condition in  $\mathcal{I}(m_i)$ . There are two cases to consider:
  - 1. The system remains in mode  $m_i$ . In this case, following the strategy for the corresponding instance of mode  $m_i$  is winning for one step, so the specification is not violated at step i + 1.
  - 2. The system transitions to a new mode  $m_j$ . A transition from mode  $m_i$  to mode  $m_j$  is allowed only if mode  $m_j$  satisfies the cumulative obligations inherited from  $m_i$ . This is ensured by an initial condition  $\theta_{s_j} \in \mathcal{I}(m_j)$  that satisfies the pending obligations from  $m_i$  (i.e.,  $\mathcal{O}^p(m_i) \cap \mathcal{O}^c(m_j)$ ). The combined strategy moves to such an initial state.

Therefore, for all steps, the combined strategy satisfies the specification.  $\Box$ 

Thm. 2 establishes soundness but not completeness. Mode-based decomposition forces the system to choose the next mode based solely on the current history, disregarding the next environment input. This can lead to false negatives. Consider the specification  $\square(m_1 \to \bigcirc(e \lor m_2))$ , where  $m_1, m_2$  are modes and e is an environment input. This requires that if the system is in  $m_1$ , then in the next step, either e is true or the system transitions to  $m_2$ . Our approach must decide on the transition to  $m_2$  before observing e. A winning strategy might depend on e: stay in  $m_1$  if e is true, and transition to  $m_2$  otherwise. For instance, if e alternates, a winning strategy is to stay in  $m_1$  while e is true and move to  $m_2$ when e is false. Our method, however, must choose between always staying in  $m_1$  (violating the spec when e is false) or always transitioning to  $m_2$  (unnecessarily when e is true), incorrectly reporting unrealizability. This demonstrates that premature decision-making can lead to false negatives. To mitigate incompleteness, we define a sufficient condition: mode-determinism, ensuring that the current variable valuation uniquely determines the next mode without violating the specification. For a  $\mathsf{GX}_0$  formula  $\Box \psi$  4 with variables  $\overline{z} = \mathit{Vars}(\psi) = \mathcal{X} \cup \mathcal{Y}$ , let  $\varphi(\overline{z}, \overline{z}')$  be the relation between pre- and post-state variables satisfying  $\varphi$ .

<sup>&</sup>lt;sup>4</sup> Nested  $\bigcirc$  operators in  $\psi$  are handled by introducing fresh variables  $v_{\bigcirc\alpha} \iff \bigcirc\alpha$ .

**Definition 6 (Mode-deterministic).** A specification  $\Box \psi$  with  $\overline{z} = Vars(\psi)$ is mode-deterministic if there are no modes  $m_1$ ,  $m_2$  and  $m_3$ ,  $m_2 \neq m_3$  s.t.: 396

$$\exists \overline{z}, \overline{z}_2, \overline{z}_3. (m_1(\overline{z}) \land (\psi(\overline{z}, \overline{z}_2) \land m_2(\overline{z}_2)) \land (\psi(\overline{z}, \overline{z}_3) \land m_3(\overline{z}_3))).$$

This condition ensures that no state in  $m_1$  can transition simultaneously to  $m_2$ 397 and  $m_3$ . Mode-determinism can be verified using  $k^3$  SAT queries, where k is the number of modes. For a mode-deterministic specification  $\varphi$  the feasible mode 399 jumps  $J(\varphi) \subseteq M \times M$  are defined as:  $(m_i, m_j) \in J$  if:  $\exists \overline{z}, \overline{z}'. (m_i(\overline{z}) \land \psi(\overline{z}, \overline{z}') \land \overline{z})$  $m_i(\overline{z}')$  This captures possible mode transitions within a  $\varphi$  trace.

**Theorem 3.** Let  $\varphi$  be a mode-deterministic specification and  $(M, \prec)$  a modegraph such that  $J(\varphi) \subseteq \prec$ . Then, if the mode-based synthesis algorithm returns 403 unrealizable,  $\varphi$  is unrealizable.

It is always possible to choose a proper  $\prec$  either by computing  $J(\varphi)$  or by using the complete mode-graph. 406

#### **Empirical Evaluation** 4

We evaluate our approach around the following research questions: 408

**RQ1:** How effectively does our method compute initial conditions? 409

**RQ2:** Does our mode-based technique improve controller synthesis time com-410 pared to traditional methods?

**RQ3:** How well does our heuristic prune the search space? 412

**Specifications.** For our evaluation, we use benchmarks from [13] alongside new 414 specifications from recent work [58,49], 415 written in languages like Spectra and 416 FRET. Additionally, we introduce goal-417 conflicts and adapt each specification to 418 mode-based determinism <sup>5</sup> making them 419 unrealizable to test whether our method 420 correctly identifies those scenarios. Table 1 421 summarizes the specifications, including

Spec.	#A - #G	#M
counter(n)	2-(n+5)	2,4,7
sis-1500	2-7	3
thermostat(n)	3-4	3
cruise-fse	3-15	4
altlayer(n)	1-9	3
lift(n)	1-187	3
fret-lift	2-14	4
double-counter(n)	2-(2n + 5)	2,4,7

Table 1: Specs and Modes.

assumptions (#A), guarantees (#G), and modes (#M). 423

Setting and Evaluation. We implemented our approach in Java, leveraging the widely-used OwL library [42] for parsing and manipulating LTL formulas. For 425 verifying realizability, we used Strix [52]. The experiments were conducted on a cluster featuring Intel Xeon processors clocked at 2.6GHz and equipped with 427 16GB of RAM, running GNU/Linux. Our tool, case studies, and instructions for replicating the experiments are in our replication package <sup>6</sup>.

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<sup>&</sup>lt;sup>5</sup> using SCR methodology [35]

<sup>6</sup> https://sites.google.com/view/mode-decomposition

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Specifications		Time (s)		#Strix		Detailed time (fp)		
Name.	#M	Mon.	fp	fph	fp	fph	Proj.	Strix
counter-10	2	8	0.56	0.49	2	2	0.41	0.15
counter-14	7	timeout	1.23	1.31	7	7	0.82	0.41
counter-20	4	timeout	1.01	1.06	4	3	0.80	0.27
lift-15	3	timeout	4.03	4.00	5	4	3.44	0.59
sis-1500	3	timeout	46.40	43.00	3	2	42.84	3.92
double-counter-10	2	6	0.80	0.73	3	3	0.60	0.20
double-counter-14	7	125	3.23	2.96	13	10	2.23	1.00
double-counter-20	4	timeout	2.62	2.56	7	5	2.14	0.48
cruise-fse	4	timeout	65.37	63.89	11	9	45.86	19.51
altlayer	3	timeout	18.13	18.52	4	4	15.83	2.30
fret-lift	4	timeout	27.48	27.12	4	4	21.80	5.68
thermostat-80	3	60	29.64	29.66	3	2	27.40	2.24
thermostat-150	3	ERROR	60.53	55.41	3	2	52.68	7.85

Table 2: All experimental results (Realizable cases).

Effectiveness and Performance Evaluation. We evaluated three approaches: the monolithic method (mon), our fixpoint method (fp), and our memoization [9] fixpoint method (fph). Both fp and fph consistently outperformed the monolithic approach across various specifications (see Table 2). Moreover, the time distribution shows that Strix time is significantly lower than the projection time (Proj).

In our corpus, particularly in SCR specifications from requirements engineering, candidates are often mutually exclusive, limiting memoization's effectiveness. While it does not bypass many realizability checks, it still moderately reduces the total number of solver calls. For more complex specifications with larger search spaces and well-defined lattice structures, memoization becomes significantly more effective. By leveraging the fact that unrealizable formulas have unrealizable supersets, and realizable formulas have realizable subsets, our approach propagates results across the lattice, pruning the search space and minimizing solver calls. Tests on randomly generated formulas using Spot [26] show that our memoization approach can cut solver calls by up to 50%, leading to faster execution times. This demonstrates the potential of our approach for efficiently handling intricate specifications in mode-based synthesis. Additionally, in the context of unrealizable cases following mode-based deterministic specifications (see Table 3), we observed the same trends as in realizable cases. This consistency further underscores the robustness and reliability of our method across both realizable and unrealizable scenarios.

Impact on Synthesis Time. Our comparative analysis revealed significant differences in synthesis time between the approaches. The monolithic method consistently reached the *ten-minute* timeout on larger instances, with some cases failing due to the size of the formulas. In contrast, both *fp* and *fph* completed synthesis well within the time limits, reducing synthesis time by over 90%.

Specifications		Time (s)		#Strix		Detailed Time (fph)		
Name	#M	Mon	fp	fph	fp	fph	Proj.	Strix
counter-10	2	8	0.61	0.59	3	2	0.39	0.20
counter-14	7	timeout	4.33	2.66	21	16	1.69	0.98
counter-20	4	timeout	2.237	4.60	11	10	3.28	1.33
lift-15	3	timeout	6.92	10.93	9	7	9.50	1.44
sis-1500	3	timeout	44.72	58.19	3	3	53.58	4.62
double-counter-10	2	13	1.28	1.83	7	7	1.14	0.61
double-counter-14	7	timeout	2.52	3.18	13	10	2.32	0.84
double-counter-20	4	timeout	6.83	9.60	23	19	7.75	1.84
cru-fse	4	timeout	99.83	134.72	23	18	101.42	33.30
altlayer	3	timeout	65.40	65.45	11	10	57.77	7.68
fret-lift	4	timeout	79.80	81.80	10	10	64.18	17.61
thermostat-80	3	74	110.67	100.84	9	7	94.20	6.64
thermostat-150	3	ERROR	193.17	185.69	9	6	173.87	11.83

Table 3: All experimental results (Unrealizable cases). All cases executed using mode-determinism.

Moreover, our techniques complement state-of-the-art decomposition tools, none of which [39,29] could handle the specifications in our corpus. Frequent use of state variables or modes posed significant challenges for these tools, as noted by Mavridou *et al.* [51]. Our mode-based approaches, however, excelled in these environments, demonstrating adaptability and effectiveness where traditional methods fall short.

### 5 Conclusion and Future Work

This paper presents a novel, fully automated mode-based reactive synthesis method. Taking an LTL  $(GX_0)$  specification and a set of modes (with optional transitions) as input, our iterative algorithm efficiently searches for initial condition combinations that realize the overall specification or concludes unrealizability. This automatic search for suitable initial conditions is a key feature of our approach, simplifying the synthesis process for engineers.

A current limitation is that completeness for unrealizability requires mode-based determinism and a subsuming mode-graph, which we plan to address in future work. However, our method achieves significantly faster synthesis compared to monolithic methods, enabling more effective derivation of controllers for complex real-world specifications, as supported by a thorough empirical evaluation. Future work will also explore controller explainability and consider extensions to richer LTL fragments.

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